

DETERMINATION CONSTANTS OF 4-ELEMENT RHEOLOGICAL MODEL WITH REBOUND RESILIENCE METHOD

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Abstract. In this paper an express-method and equipment for experimental determination of the 4-element rheological model constants (2 elastic modulus, 2 viscosity coefficients) for elastomers are developed. The method is based on rebound of falling hammer against an elastomer material. A mathematical model for describing this collision is elaborated and an algorithm for determination of the constants of the model was developed and realized in Matlab software. Time necessary for determination of the constants is until 5 minutes. It gives an opportunity to significantly accelerate the design and production process of elastomer-metal vibration isolator prototypes with defined parameters of stiffness and damping. The method allows to trace displacement, velocity and acceleration of the metal parts of the shock absorber throughout the collision; this may be realized with Matlab or an analogical program of numerical integration. For forecasting of the mechanical properties of the shock absorber during a long period (considering the aging of elastomer) it is necessary to know the rheological model parameter dependence on time, temperature, energy input and other factors; the proposed method allows to significantly accelerate these experimental studies.

Keywords: elastomer, rheological model, elastic modulus, viscosity coefficient, rebound resilience.

Introduction

Elastomers as products or components are widely used in various industry sectors, such as mechanical engineering, construction, industry of medical equipment, manufacturing of footwear, etc. Elastomers are subjected to static, dynamic and cyclic loading, they dampen shocks and vibrations. Elastomeric product design is associated with a number of specific properties – volume incompressibility, creep, stress relaxation, aging [1; 2]. Many authors considered in their works the elastomeric product design, for example, calculations of vibroinsulation elements [3; 4], static loadings at small and large deformations [1; 2] damping elements in construction [4], calculations of thin-layer elastomer-metal dampers [5; 6], shock damping properties of shoe soles [7], elastomer lifetime forecasting and aging [1; 8-10], acoustic absorbers [11], elastomeric actuators for robotic fish [12]. In order to correctly design elastomer products, it is necessary to choose the appropriate model and to know the values of the model constants (physical parameters of materials). As one of the elastomer calculation variants is rheological models [1; 13; 14]. For description of the slow ongoing process (creep, stress relaxation), the 4-element model is considered the best [3]. The given model includes 4 parameters, characterizing the mechanical properties of elastomers – 2 modules of elasticity E_1 , E_2 , and 2 viscosity coefficients η_2 un η_3 . The number of elastomer types, available on the market, is huge. Nowadays, elastomers may also be made of two components of liquids, their properties may be modified using a variety of fillers including copper nanoparticles [15], silica nanoparticles [16], recycled raw materials - waste tire particles [11], etc. In addition, the material over time is aging, changing its properties. This means that in each specific case, the material constants will be different. Thus, the best option is to determine the parameters of elastomers experimentally every time. This requires a simple, fast and sufficiently accurate method. In the work [17] the method of determination of constants for 4 elements rheological model using the creep experiment at constant tensile stress and at constant tensile forces is viewed. Unfortunately, the creep experiment requires a significant expenditure of time (at least 2 hours), because there is a need to measure the permanent deformations. The work [13] sets the criteria for the determination of permanent deformations; these deformations are measured within 1h after removal of the load. It is technically difficult to maintain a constant tensile stress during the creep test [9], [10]. It is easier to ensure a constant tensile force, but in this case to determine the constants it is necessary to calculate the integral

$$\int_{t_1}^{t_2} \varepsilon(t) dt$$

from experimental curves $\varepsilon = \varepsilon(t)$, where ε – deformation, t – time. This makes determination of constants more difficult and complicates automation of the measurement process. In this work the creep experiment is replaced by the rebound elasticity test method [18].

The aim of the given work is to develop an express method (algorithm) for experimental determination of constants (2 elastic modules, 2 viscosity coefficients) of the 4-element rheological model of elastomeric materials.

Materials and methods

The constants of the rheological model are experimentally determined for Easy Composites Ltd Xencast PX30 soft flexible polyurethane rubber. The method is also valid for any other elastomer. The samples of material have a cylindrical shape of 40 mm diameter, 12 mm height and with mass 15.4 g. The steel disc with mass $m_2 = 51.4$ g, diameter 41 mm and thickness 5 mm is glued to the front base of a rubber cylinder. Rear base of the cylinder is glued to the side surface of heavy (mass $m_3 = 25$ kg) parallelepiped. Mass $m_1 = 308$ g (i.e. hammer of pendulum in bearings [18]) at a speed of v_0 impacts the steel - elastomer shock absorber, which is firmly fixed to the mass m_3 (Fig. 1, 2). It is assumed that $m_3 \gg m_1 > m_2$, the arm $L = 200$ mm of the hammer is long, its mass and elastomer mass are negligibly small. As the elastomer layer of the shock absorber brakes the movement of mass m_2 in the x-axis direction, then after the collision of both bodies m_1 and m_2 continue to move together. The speed v of shock absorber plate m_2 immediately after the collision with the mass m_1 may be calculated using the momentum conservation law:

$$v = \frac{v_0 \cdot m_1}{m_1 + m_2}. \quad (1)$$

This moment of time considered as the beginning; when the time $t = 0$ mass m_1 and m_2 have the speed v , displacement of the front plate m_2 of the shock absorber $u = 0$. For the description of the properties of elastomers the 4-element rheological model is used, which consists of 2 elastic elements with stiffness c_1 and c_2 (the elastic modules E_1 and E_2) and 2 damping elements with coefficients of viscosity η_2 and η_3 [8], [9]. The model is described with a system of equations:

$$\begin{cases} \sigma = E_1 \cdot \varepsilon_1 \\ \sigma = E_2 \cdot \varepsilon_2 + \eta_2 \cdot \dot{\varepsilon}_2 \\ \sigma = \eta_3 \cdot \dot{\varepsilon}_3 \\ \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \end{cases}, \quad (2)$$

where σ – mechanical stress applied to elastomer, Pa;

ε – engineering normal strain of the elastomer layer (total strain);

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ – relative longitudinal deformation (strain) of elements E_1, E_2, η_2 and η_3 .

Assume that at an arbitrary time moment t displacement of mass m_2 is equal u , absolute longitudinal deformation of element E_1 is u_1 , displacement of parallel connected elements – E_2 and η_2 is u_2 , of element η_3 – is u_3 . Having regard to the second Newton's law, Hooke's law, definitions of the force of viscosity, stress and relative longitudinal deformation, the following mathematical model for description of the given process is received:

$$\begin{cases} (m_1 + m_2 + m_{el}) \cdot \ddot{u} = -c_1 \cdot u_1 \\ (m_1 + m_2 + m_{el}) \cdot \ddot{u} = -c_2 \cdot u_2 - \eta_2 \cdot \dot{u}_2 \cdot \frac{S_0}{l_0} \\ (m_1 + m_2 + m_{el}) \cdot \ddot{u} = -\eta_3 \cdot \dot{u}_3 \cdot \frac{S_0}{l_0} \\ u = u_1 + u_2 + u_3 \end{cases}, \quad (3)$$

where S_0 – cross-sectional area of elastomer (perpendicular to the impact load), m^2 ;

l_0 – thickness of the elastomer layer at the beginning, m;
 m_{el} – mass of the elastomer layer, kg;
 c_1, c_2 – stiffness constants, $N \cdot m^{-1}$:

$$c_1 = E_1 \cdot \frac{S_0}{l_0}, \quad c_2 = E_2 \cdot \frac{S_0}{l_0}. \tag{4}$$

The model (3) has 4 equations and 4 unknown functions $u_1 = u_1(t)$, $u_2 = u_2(t)$, $u_3 = u_3(t)$, $u = u(t)$. Time derivative of the last equation in system (3) gives the relationship between all speeds of elements:

$$v = v_1 + v_2 + v_3, \tag{5}$$

where $v = v(t)$ – movement speed in the x-axis direction for the front plate m_2 of the shock absorber, $m \cdot s^{-1}$;
 $v_1 = v_1(t)$ – deformation speed of the element E_1 ;
 $v_2 = v_2(t)$ – deformation speed of the elements E_2 and η_2 ;
 $v_3 = v_3(t)$ – deformation speed of the element η_3 .

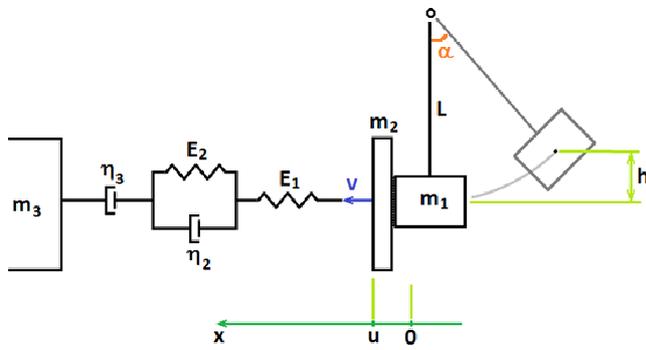


Fig. 1. Calculation model

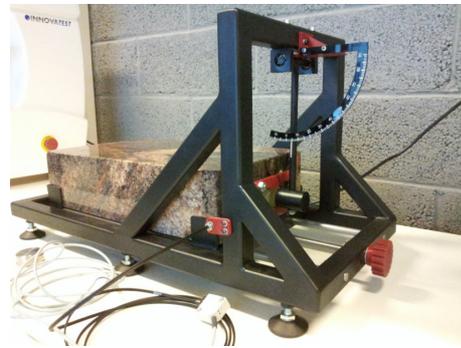


Fig. 2. Schob- pendulum

Solving of equation system (3) is realized numerically by small time steps Δt , which are numbered consecutively: $i = 1, 2, 3 \dots$. Derivatives in the system of equations (3) are replaced with forward differences of displacement and speed in the time interval Δt :

$$\begin{cases} (m_1 + m_2 + m_{el}) \cdot \frac{\Delta v}{\Delta t} = -E_1 \cdot \frac{S_0}{l_0} \cdot u_1 \\ (m_1 + m_2 + m_{el}) \cdot \frac{\Delta v}{\Delta t} = -E_2 \cdot \frac{S_0}{l_0} \cdot u_2 - \eta_2 \cdot \frac{\Delta u_2}{\Delta t} \cdot \frac{S_0}{l_0}, \\ (m_1 + m_2 + m_{el}) \cdot \frac{\Delta v}{\Delta t} = -\eta_3 \cdot \frac{\Delta u_3}{\Delta t} \cdot \frac{S_0}{l_0} \end{cases} \tag{6}$$

Initial conditions: if $t = 0$ then

$$\begin{cases} u = 0, \quad v = \frac{v_0 \cdot m_1}{m_1 + m_2}, \\ u_1 = 0, \quad v_1 = v, \\ u_2 = 0, \quad v_2 = 0, \\ u_3 = 0, \quad v_3 = 0. \end{cases} \tag{7}$$

After collision the mass m_1 and m_2 continues to move along the x-axis direction and compresses the elastomer layer of the shock absorber. This kinetic energy is converted into elastic potential energy

of elastomer deformation (accumulating in elements E_1 and E_2) and into heat (released in elements η_2 and η_3). Mass m_1 and m_2 are braked, then they start accelerated motion in the opposite direction of the x -axis. In elements E_1 and E_2 accumulated elastic potential energy is converted into kinetic energy of mass m_1 and m_2 and in heat (released in element η_2). At time $t = t^*$ elastic deformation of element E_1 becomes zero, the mass m_1 has gained the speed v^* and separated from the mass m_2 . Then element E_1 starts to stretch out and begins to brake the mass m_2 . End conditions: if $t = t^*$, then

$$\begin{cases} u_1 = 0, \\ v = v^*. \end{cases} \quad (8)$$

After the rebound, the mass m_1 rises at a height h . From the principle of conservation of mechanical energy it follows that the mass m_1 speed immediately after the rebound from the shock absorber is:

$$v^* = \sqrt{2 \cdot g \cdot h} , \quad (9)$$

where $g = 9.81 \text{ m} \cdot \text{s}^{-2}$; friction is disregarded.

Results and discussion

The numerical calculation algorithm includes the following steps.

1. the initial experimental data collection: 4 measurements are carried out – the hammer m_1 of pendulum falling with 4 different heights $h_{10}, h_{20}, h_{30}, h_{40}$, bounces off from the shock absorber and takes off a height h_1, h_2, h_3, h_4 (see Fig. 1); experimental values of the speeds $v_{01}, v_{02}, v_{03}, v_{04}$ of the hammer m_1 are calculated shortly before the collision with the shock absorber and the speeds $v^*_{\text{exper}1}, v^*_{\text{exper}2}, v^*_{\text{exper}3}, v^*_{\text{exper}4}$ immediately after the rebound:

$$v_{0j} = \sqrt{2 \cdot g \cdot h_{0j}} , \quad v^*_{\text{exper}_j} = \sqrt{2 \cdot g \cdot h_j} , \quad (10)$$

where $j = 1, 2, 3, 4$ – numbers of experimental measurement;

m_1, m_2 – masses are measured;

2. deformation rate v_{1j} of element E_1 and the speed v_j of the plate m_2 of the shock absorber at initial moment ($t = 0$):

$$v_{1j} = \frac{v_{0j} \cdot m_1}{m_1 + m_2} , \quad v_j = \frac{v_{0j} \cdot m_1}{m_1 + m_2} ; \quad (11)$$

3. value range (minimum and maximum value) of parameters E_1, E_2, η_2, η_3 and the step of changes are defined; in computer program of numerical calculation 4 cycles are created, in which these parameters change (4 cycles of one in another). Determination of the parameters E_1, E_2, η_2, η_3 is based on 4 experimental measurements with numbers $j = 1, 2, 3, 4$, therefore the 5 cycle with variable j is created inside these 4 cycles:

$$\begin{aligned} & \left[\begin{array}{l} \text{for } E_1 = E_{1\min} \text{ to } E_{1\max} \text{ step } \Delta E_1 \\ \left[\begin{array}{l} \text{for } E_2 = E_{2\min} \text{ to } E_{2\max} \text{ step } \Delta E_2 \\ \left[\begin{array}{l} \text{for } \eta_2 = \eta_{2\min} \text{ to } \eta_{2\max} \text{ step } \Delta \eta_2 \\ \left[\begin{array}{l} \text{for } \eta_3 = \eta_{3\min} \text{ to } \eta_{3\max} \text{ step } \Delta \eta_3 \\ \left[\begin{array}{l} \text{for } j = 1 \text{ to } 4 \\ \quad 4 - 14\text{th step of algorithm} . \\ \end{array} \right] \\ \text{end} \end{array} \right] \quad (12) \end{aligned}$$

4. deformation change of element E_1 in time interval Δt :

$$\Delta u_1 = v_1 \cdot \Delta t, \quad (13)$$

where $v_1 := v_{1j}$;

“:=” – assignment operator;

5. deformation of element E_1 at time moment t :

$$u_1 := u_1 + \Delta u_1; \quad (14)$$

6. from the first equation of the system (6) the speed change of the plate m_2 at time interval Δt is obtained:

$$\Delta v = - \frac{E_1 \cdot S_0 \cdot u_1}{(m_1 + m_2 + m_{el}) \cdot l_0} \cdot \Delta t; \quad (15)$$

7. speed of the plate m_2 of the shock absorber at time moment t :

$$v := v + \Delta v; \quad (16)$$

8. from the equations system (6) (first and second equation) the deformation of elements E_2 and η_2 at time moment $t = \Delta t \cdot i$ is obtained:

$$u_{2i} = \frac{E_1 \cdot u_1 + \frac{\eta_2}{\Delta t} \cdot u_{2(i-1)}}{E_2 + \frac{\eta_2}{\Delta t}}, \quad (17)$$

where $i = 1, 2, 3, \dots$ – number of time interval Δt ;

9. deformation speed of elements E_2 and η_2 at time moment t :

$$v_2 = \frac{u_{2i} - u_{2(i-1)}}{\Delta t}; \quad (18)$$

10. from the equations system (6) (first and third equation) the deformation of element η_3 is obtained for time moment t :

$$u_{3i} = \frac{E_1 \cdot u_1 + \frac{\eta_3}{\Delta t} \cdot u_{3(i-1)}}{\eta_3} \cdot \Delta t; \quad (19)$$

11. deformation speed of element η_3 at time moment t :

$$v_3 = \frac{u_{3i} - u_{3(i-1)}}{\Delta t} \quad (20)$$

12. deformation speed of element E_1 at time moment t :

$$v_1 = v - v_2 - v_3; \quad (21)$$

13. displacement and acceleration of plate m_2 of the shock absorber at time moment t :

$$u = u_1 + u_2 + u_3, \quad (22)$$

$$a = \frac{\Delta v}{\Delta t}; \quad (23)$$

14. if $u_1 > 0$ then $t := t + \Delta t$ and return to step 4 of algorithm;
 if $u_1 \leq 0$ and $j < 4$ then end of calculation with the given j ;
 $t_j^* := t$ and $v_j^* := v$ are assigned values; transition to next j and to step 4 of algorithm;

if $u_1 \leq 0$ and $j = 4$ then end of calculation with the given E_1, E_2, η_2, η_3 ; assigned values $t_4^* := t$ and $v_4^* := v$; the root-mean-square deviation (RMSD) between 4 calculated speeds and 4 experimentally obtained speeds after the rebound is determined:

$$s = \sqrt{(v^*_1 - v^*_{\text{exp } er1})^2 + (v^*_2 - v^*_{\text{exp } er2})^2 + (v^*_3 - v^*_{\text{exp } er3})^2 + (v^*_4 - v^*_{\text{exp } er4})^2}; \quad (24)$$

if $s < s_{\text{min}}$, then $s_{\text{min}} := s$ and the values of appropriate parameters E_1, E_2, η_2, η_3 are optimal; assigned values $E_{1\text{opt}} := E_1, E_{2\text{opt}} := E_2, \eta_{2\text{opt}} := \eta_2, \eta_{3\text{opt}} := \eta_3$ (s_{min} – minimal value of RMSD; at the beginning of calculation $s_{\text{min}} :=$ arbitrary large number); go to the next values E_1, E_2, η_2, η_3 ;

15. after execution of all cycles the optimal values of the parameters are determined: $E_{1\text{opt}}, E_{2\text{opt}}, \eta_{2\text{opt}}, \eta_{3\text{opt}}$; this is a search result.

The given algorithm is practically implemented in MATLAB program. The experimental data are obtained using Schob - pendulum (Fig. 2) [18]. For Easy Composites Ltd Xencast PX30 soft flexible polyurethane rubber the following values of constants are received: $E_1 = 4.93 \pm 0.24$ MPa, $E_2 = 0.562 \pm 0.054$ MPa, $\eta_2 = (1.77 \pm 0.073) \cdot 10^4$ Pa·s, $\eta_3 = (6.18 \pm 0.60) \cdot 10^7$ Pa·s; elastic modulus at static load $E = 3.20 \pm 0.10$ MPa (obtained by compression test with machine Zwick / Roell Z-150).

In the rebound elasticity test for the given material it is established: if the maximum tilt angle α of Schob pendulum before the collision is $48^\circ \pm 1^\circ$, then after the collision it is $29^\circ \pm 0.5^\circ$. Using the obtained constants E_1, E_2, η_2, η_3 values and the given algorithm, displacement, absolute deformation, speed and acceleration values during the impact were numerically calculated, in Fig. 3-6 the plots of dependence of these values on the time are presented. From the given plots the end moment of collision may be determined; it conforms to the time $t = 0.00265$ s, absolute deformation is $u_1 = 0, u_2 = \text{max}$ (maximum value), $u_3 = \text{max}$, the velocity $v = \text{max}$ and acceleration $a = 0$.

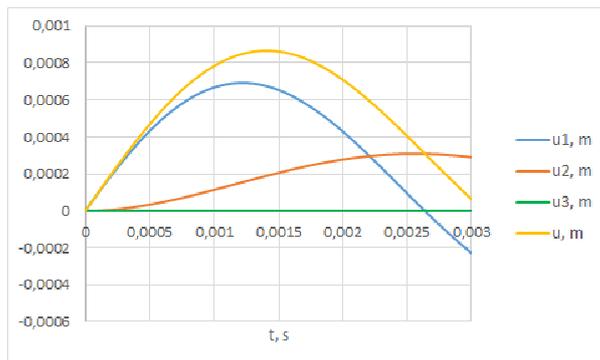


Fig. 3. Displacements u and absolute deformations u_1, u_2, u_3 during the impact

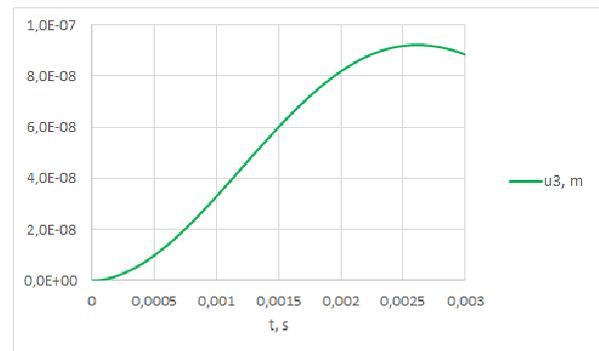


Fig. 4. Absolute deformation u_3 during the impact

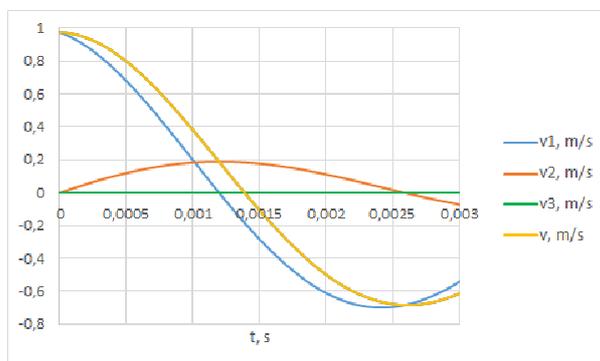


Fig. 5. Deformation speeds v, v_1, v_2, v_3 during the impact

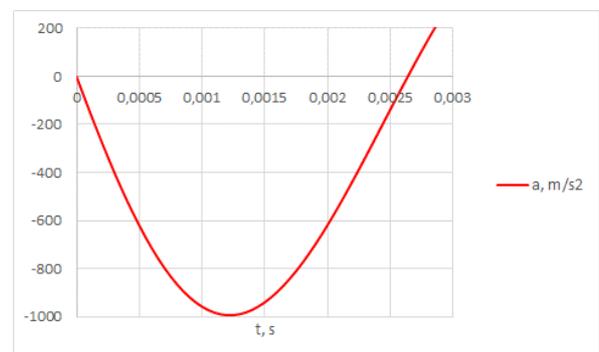


Fig. 6. Acceleration the of plate m_2 of the shock absorber during the impact

Conclusions

1. The proposed method gives the possibility of significantly shortening the time of designing the metal-elastomer dampers with the required stiffness and damping characteristics, because it allows fast (within 5 minutes) definition of the constants E_1 , E_2 , η_2 , η_3 of elastomer.
2. The method allows to trace displacement, velocity and acceleration of the metal parts of the shock absorber throughout the collision; this may be realized by numerical integration using Matlab or an analogical program.
3. For forecasting of the mechanical properties of the shock absorber during a long period (considering the aging of elastomer) it is necessary to know the dependence of the rheological model parameters on the time, temperature, energy input and other factors; the given method allows to accelerate significantly these experimental studies.
4. In future investigation it is planned to compare the constants E_1 , E_2 , η_2 , η_3 values obtained with help of the proposed method and using the creep test.

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